Some New Results on the H Dibaryon in the Quark Cluster Model

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Abstract

The H dibaryon channel, $(I = 0, J = 0, S = -2)$, is revisited in the non-relativistic quark cluster model (NRQCM) using a basis extended beyond the usual set of baryon cluster pairs to include an explicit spatially symmetric 6q state, analogous in structure to the MIT bag model H. We find that the binding predicted using the two-baryon basis alone is significantly deepened by the addition of the additional 6q configuration. The NRQCM thus appears, contrary to earlier findings, to be incompatible with the experimental information available for this channel.

12.39.Jh, 12.39.Mk, 12.39.Pn, 14.20.Pt

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Since Jaffe's original observation that, in the MIT bag model, a deeply bound (on a nuclear scale) strong interaction stable di-baryon state, the H, is expected in the $(I, J, S) = (0, 0, -2)$ channel [1], considerable theoretical and experimental attention has been focussed on this channel. In addition to numerous experimental searches (see Ref. [2] for a recent review of searches for doubly strange systems, including the H, as well as other references cited therein), a variety of models have been employed to study the H channel [3–6]. In most, though not all of these models, a deeply bound H also occurs. It is important to note that, although all these models are constrained by the baryon sector, and generally capable of being parametrized so they are quite successful there, there is no obvious reason to assume that, so parametrized, they must necessarily be equally successful in the dibaryon sector. This observation follows trivially from that fact that the colour **6** qq configurations, unavoidably present in the dibaryon sector, are not present in the single baryon sector, and hence have interactions unconstrained by single baryon phenomenology. Only to the extent that the models are actually successful in capturing all of the relevant features of the underlying field theory (QCD) would such an extension be justified. As such, experimental searches for the H have the potential payoff of either providing highly non-trivial support for the extension of these models to the multi-quark sector, or of unambiguously demonstrating that they have failed to model features of QCD crucial to a proper treatment of that sector.

Among existing experimental searches, the most constraining to date is the unambiguous observation of the sequential Λ decay of a double Λ hypernucleus [7]. If the H is deeply bound (on the scale of the relevant nuclear binding energies) then the *strong* decay $_{\Lambda\Lambda}^{Z}(A+2) \rightarrow H + X$ should totally dominate any sequential weak decay process. Although the two-body process $_{\Lambda\Lambda}^{Z}(A+2) \rightarrow H + {^{Z}A}$ can receive strong kinematic suppression for a very deeply bound H [8], for a nucleus of modest size (such as those which are candidates for the sequential Λ decay event) many inelastic channels should be present, for which this suppression would not exist. The observed sequential Λ decay thus limits the H binding to be less than 28 MeV (if the candidate double Λ hypernucleus is taken to be $_{\Lambda\Lambda}^{13}B$, less if the alternate assignment $_{\Lambda\Lambda}^{10}Be$ is accepted).

As just explained, the sequential Λ decay observation immediately rules out many existing models, at least as applied to the multi-quark sector. One model which apparently survives, however, is the quark cluster model. This observation, if true, is doubly interesting because, as is well known, the qq colour hyperfine interaction of the model very naturally provides a residual exchange-induced repulsion having the correct relative strengths in the various NN and YN S-wave channels (see Refs. [9,10] and references cited therein). Existing calculations [4,6] which employ the usual one-gluon-exchange effective interaction [13], supplemented by long- and intermediate-range meson exchange forces (π and σ exchange) with σ couplings arranged to provide fits to NN and YN scattering data, find, for the H channel, (a) no binding, but a sharp channel coupling effect in $\Lambda\Lambda$ scattering associated with the opening up of the NΞ threshold, in the absence of the additional meson exchange forces [4] (b) weak binding, of order 10-20 MeV, when meson exchange forces are included [6]. Such results are, of course, compatible with existing experimental observations.

The actual situation for the cluster model, however, is not so clear as might appear from the discussion above. Although in the S-wave NN and YN channels one can show that localized 6q configurations play little role (see Refs. [9,14] and references cited therein), this is not necessarily the case for all two-baryon channels. The physics of this statement is rather simple to understand. In the S-wave NN and YN channels, the colour hyperfine interaction provides a strong residual repulsion which shields the two-baryon channels from access to 6q states localized at short distances. Two-baryon channels which happen not to have a strong repulsion in the model, however, need not have such a suppression of localized configurations, and the effective absence of the coupling of such configurations to the S-wave NN and YN channels does not, therefore, necessarily argue for the absence of such couplings to all BB$'$ channels. In fact, an example is known which precisely demonstrates this point. In the $(I, J, S) = (0, 3, 0)$ channel (whose lowest-lying relative S-wave BB$'$ channel is $\Delta\Delta$), the residual $\Delta\Delta$ interaction in the model (without meson-exchange forces) is weakly attractive, producing a binding of a few MeV [15,16]. Adding an available localized configuration, however, increases the binding dramatically, to 260 MeV [16]. This observation is especially

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relevant in view of the fact that the residual NΞ interaction in the model is known to be attractive [4], this channel, therefore, providing a potential 'entry' channel for significant coupling to localized 6q configurations.

In this paper we investigate the question of whether localized 6q configurations might play a significant role in the H channel in the quark cluster model by adding to the ΛΛ,ΣΣ, NΞ basis of previous investigations, a localized 6q configuration modelled on the MIT bag model H. This by no means exhausts the potentially important localized configurations, but is a natural starting point in view of the fact that the bag model (spatially symmetric, colour-spin-flavour antisymmetric) configuration is known to have optimally attractive discrete hyperfine expectation [1]. We consider the model both in the presence and absence of this state (let us, for conciseness, call it h) and in both the presence and absence of the long- and intermediate-range meson exchange forces. In addition, we study the sensitivity of the predictions to small variations in some of the model parameters. This latter point is of some importance since, for example in the MIT bag model, it is known that the existence or non-existence of binding depends extremely sensitively on the precise value of the bag model vacuum energy parameter $B^{1/4}$ [17]. In general one needs to verify that the model predictions obtained are robust, in the sense of remaining qualitatively and perhaps semi-quantitatively unchanged under small variations of the parameters such as might be anticipated in attempting to extend an effective model to a slightly different physical and kinematic realm than that in which it was originally parametrized.

Let us now briefly outline our calculation. As mentioned above, we extend the basis consisting of ΛΛ, ΣΣ, and NΞ two-baryon states used in previous calculations to include an analogue, h, of the MIT bag-model H. The state h must be spatially symmetric, have overall $(I, J, S) = (0, 0, -2)$ quantum numbers, and be antisymmetric in the combination of spin, flavour, and colour, which restrictions lead to the form

$$h = N_h \ \exp\left(-\frac{b_H^2}{2}\sum_{i=1}^{6}(\vec{r_i} - \vec{R}_{cm})^2\right)|h\rangle_{JFC} \tag{1}$$

where $\vec{r_i}$ is the coordinate of the i-th quark, $\vec{R}_{cm}$ is the center of mass coordinate, N_h is a

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normalization factor, and $|h\rangle_{JFC}$ represents the discrete spin-flavour-colour part of the state. The latter is constructed starting from the assumption that the H is *very nearly* a **490** in colourspin; as shown in Ref. [5], this is correct to first order in the flavour symmetry breaking (details of this construction will be given elsewhere [18]). We fix the size parameter, b_H, in such a way as to reproduce the MIT bag ratio of 3q to 6q cluster sizes.

The Hamiltonian used in this study includes cluster model short-range potentials as well as scalar and pseudoscalar meson exchange parts, needed to reproduce medium and long-range behaviour, which we adapt from Refs. [11,12],

$$H = T + V_c + V_{OGE} + V_\sigma + V_\pi + V_K. \tag{2}$$

As usual, the kinetic, confinement, and one-gluon-exchange (OGE) pieces are given by

$$T = \sum_i \frac{p_i^2}{2\overline{m}_q} - T_{cm} \tag{3}$$

$$V_c = -a_c \sum_{i<j} (\lambda_i \cdot \lambda_j) r_{ij} \tag{4}$$

$$V_{OGE} = \frac{\alpha_s}{4} \sum_{i<j} (\lambda_i \cdot \lambda_j) \left[\frac{1}{r_{ij}} - \frac{\eta_{ij}}{\beta_i \beta_j \overline{m}_q^2} \left(1 + \frac{2}{3}(\sigma_i \cdot \sigma_j) \right) \delta^3(r_{ij}) \right]. \tag{5}$$

repectively, where $\overline{m}_q$, the average quark mass, is taken to be one-third of the average octet baryon mass, a_c, the linear confinement strength, is fixed to stabilize the 3q cluster against small changes in size, and α_s, the effective strong coupling constant, is fixed (in combination with contributions due the qq one pion exchange, where present) to reproduce the N $-$ Δ mass difference. In V_{OGE}, the function η_{ij} is used to introduce flavour symmetry breaking as in Ref. [4]; it takes on one of three values (η_{ll}, η_{ls}, η_{ss}), depending on the combination of quark flavours (light or strange) in the pair ij. The function β_i is used to scale the average quark mass in those sectors where symmetry breaking is taken into account. In the presence of mesons the function β_i is adjusted to yield a light quark mass of 313 MeV and a strange quark mass of 515 MeV (i.e. $m_i = \beta_i \overline{m}_q$) [12].

The meson exchange potentials, taken from Ref. [12], include a form factor at the quark meson interaction vertex and have the form

$$V_{ps}(r_{ij}) = \frac{1}{3}\alpha_{ch}\frac{\Lambda_{CSB}^2}{\Lambda_{CSB}^2 - m_{ps}^2}\left[\frac{e^{-m_{ps}r_{ij}}}{m_{ps}r_{ij}} - \left(\frac{\Lambda_{CSB}^3}{m_{ps}^3}\right)\frac{e^{-\Lambda_{CSB}r_{ij}}}{\Lambda_{CSB}r_{ij}}\right]\sigma_i\cdot\sigma_j\,\widehat{O}_{ij}^F \tag{6}$$

$$V_\sigma(r_{ij}) = -\alpha_{ch}\frac{4\overline{m}_q^2\beta_i\beta_j}{m_\pi^2}\frac{\Lambda_{CSB}^2}{\Lambda_{CSB}^2 - m_\sigma^2}m_\sigma\left[\frac{e^{-m_\sigma r_{ij}}}{m_\sigma r_{ij}} - \left(\frac{\Lambda_{CSB}}{m_\sigma}\right)\frac{e^{-\Lambda_{CSB}r_{ij}}}{\Lambda_{CSB}r_{ij}}\right]. \tag{7}$$

Here m_σ and m_π are the σ meson and pion masses, Λ_{CSB} is the chiral symmetry breaking scale, and $\widehat{O}_{ij}^F$ is the SU(3) flavour operator ($\widehat{O}_{ij}^F = \sum_{k=1}^3 \lambda_k^F(i)\lambda_k^F(j)$ for pion exchange, and $\widehat{O}_{ij}^F = \sum_{k=4}^7 \lambda_k^F(i)\lambda_k^F(j)$ for kaon exchange). The chiral coupling constant is given by

$$\alpha_{ch} = \left(\frac{3}{5}\right)^2\frac{g_{\pi NN}^2}{4\pi}\frac{m_{ps}^2}{4M_B^2}e^{(-\frac{1}{3}m_{ps}^2 r_B^2)} \tag{8}$$

where r_B is the rms baryon cluster size and M_B is the average octet baryon mass. Since our primary aim is to explore the potential role of localized 6q configurations, we have, for simplicity, omitted contributions associated with η exchange, which are known to be small [6], and introduced flavour symmetry breaking only in the colour hyperfine and σ exchange potentials. We also drop any tensor terms since none of our basis states contain orbital excitations.

Given the above model Hamiltonian, we perform a bound-state resonating group method (RGM) calculation for the six-quark system. As usual the RGM ansatz is that the two-baryon basis states are of the form

$$BB' = \mathcal{A}[\phi_B(\tau_B)\phi_{B'}(\tau_{B'})\chi(\vec{R}_{BB'})], \tag{9}$$

where $\phi_B(\tau_B)$ is the wavefunction of a three-quark cluster in terms of its internal degrees of freedom τ_i, and the function $\chi(\vec{R}_{BB'})$ is the variational degree of freedom. A single variational parameter, c_H say, then describes the admixture of the state h into the final wavefunction.

As stated earlier, we have studied the 6q system under two different versions of the Hamiltonian (2) and in both cases examined the effect of adding the h to the two-baryon

basis. First we re-visited the Hamiltonian of Ref. [4] (call it $\mathcal{H}_1$) which includes only the short range (OGE + confinement) potentials, and then we added the σ, π, and K exchange potentials of Ref. [12] as described above (call this new Hamiltonian $\mathcal{H}_2$). The parameter values corresponding to each case are given in Table 1. The flavour symmetry breaking parameters, η_{ls} and η_{ss}, are those of Ref. [4] in $\mathcal{H}_1$, while for $\mathcal{H}_2$ the values indicated in Table 1 are derived based on the OGE interaction parameters found in Ref. [12], as are the remaining meson exchange parameters. Throughout this study $\eta_{ll} = 1$ as α_s is fixed in the light quark sector.

Our results may be summarized as follows: 1) using $\mathcal{H}_1$, in the absence of the h, the NRQCM without meson exchange does not support a bound dibaryon state, consistent with the findings of Ref. [4]. 2) Again using $\mathcal{H}_1$, the addition of the h basis state induces a weakly bound state with a binding energy of 6 MeV. 3) Using $\mathcal{H}_2$, we find a weak binding energy of 24 MeV which is consistent with previous reports of weak binding in Ref. [6]. (Note that, in Ref. [6], a phenomenological σ-baryon interaction was used, in contrast to the effective σqq interaction employed here. The latter was necessitated, in our work, by the presence of the h, since the h channel cannot be represented by two well-separated baryons). 4) Again using $\mathcal{H}_2$, the addition of the h now induces a very deeply bound state with a binding energy of 252 MeV.

The model used here includes a few parameters which are not well constrained, so it is important to understand how the results presented above might change for small variations in those parameters. For example if we allow for $\pm 10\%$ changes in the mass of the σ meson we find that the binding energy, in the presence of h, can change by as much as 15 MeV (inversely with m_σ). The results also display significant sensitivity to the quark mass and to the h size parameter b_H^{-1}, though in all cases the binding energy is far greater than the experimental upper bound. Since none of these uncertainties affect the qualitative nature of our conclusions, we defer the detailed discussion to a later work [18].

In summary, we have studied the H channel in the NRQCM using a basis of states consisting of the usual two-baryon configurations plus a 6q state, h, analogous to the bag

model H. The admixture of this state is found to yield a weakly bound state when only confinement and OGE effects are included, and in the presence of scalar and pseudoscalar meson exchange interactions it is found to induce a very deeply bound H dibaryon solution. Both observations are at odds with previous reports of no binding (confinement +OGE) or weak binding (confinement + OGE + meson exchange). This result should serve to emphasize the care that must be taken in truncating the possible set of basis states in the multiquark sector. This is particularly important in cases where the diagonal potential in the conventional BB' basis is attractive, thus providing a potential entry channel to the more localized configurations. In the context of recent experimental observations of double-Λ hypernuclear decay mentioned earlier these results also suggest that, like many other QCD-based models, the quark cluster model fails to reproduce 6q phenomenology in the H channel.

TABLES

TABLE I. Model parameters - based on Ref. [4] for $\mathcal{H}_1$, and on Ref. [12] for $\mathcal{H}_2$.

	$\overline{m}_q$ (fm^{-1})	r_B (fm)	β_l	β_s	η_{ls}	η_{ss}	b_H^{-1} (fm)	Λ_{CSB} (fm^{-1})	m_π (fm^{-1})	m_σ (fm^{-1})	m_N (fm^{-1})	$\frac{g^2_{\pi NN}}{4\pi}$	α_s
$\mathcal{H}_1$	1.944	0.5	1.0	1.0	0.6	0.1	0.56	-	-	-	-	-	1.322
$\mathcal{H}_2$	1.944	0.55	0.815	1.342	0.9	0.725	0.62	4.2	0.7	3.42	5.83	14.8	0.74

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